MOTION IN A STRAIGHT LINE

INTRODUCTION

In this chapter, we shall learn how to describe motion **(Kinematics)** without going into the causes of motion **(Dynamics).** For this, we develop the concepts of velocity and acceleration. We shall confine ourselves to the study of motion of objects along a straight line, also known **as rectilinear motion**.

For the case of rectilinear motion with **uniform acceleration**, a set of simple equations can be obtained. Finally, to understand the relative nature of motion, we introduce the concept of **relative velocity.**

In our discussions, we shall treat the objects in motion as **point objects**. This approximation is valid so far as the size of the object is much smaller than the distance it moves in a reasonable duration of time.

Motion:
An object is said to be in motion if it changes its position w.r.t. its surroundings with the passage of time .e.g. A train is moving on rails
Rest:
If an object does not change its position with respect to its surroundings with time, then it is called at rest.
[Rest and motion are relative states. It means an object which is at rest in one **frame of reference** can be in motion in another frame of reference at the same time.]
Types of Motion:
1. One Dimensional Motion (Linear Motion or Rectilinear Motion)
If only one out of three coordinates specifying the position of the object changes with respect to time.
Ex. motion of a block in a straight line, motion of a train along a straight track etc.
2. Two-Dimensional Motion:(Motion in a Plane)
If only two out of three coordinates specifying the position of the object changes with respect to time.
Ex. A circular motion etc
3. Three-Dimensional Motion:(Motion in space)
If all the three coordinates specifying the position of the object changes with respect to time,
Ex. A few instances of three dimension are flying bird, a flying kite, a flying aero plane etc.
Types of Linear Motion
The two types of linear motion can be stated as follows:

1. Uniform linear motion
   A body is known to be in uniform motion if it covers equal distance in equal motion time-span.
   Here, the motion is with zero acceleration and constant velocity.

2. Non-Uniform linear motion
   Whereas, a body is known as non-uniform if it covers unequal distance in equal time-span.
   It comprises with non-zero acceleration and variable velocity

Distance:
The length of the actual path traversed by an object is called the distance.
It is a scalar quantity and it can never be zero or negative during the motion of an object.
Its SI unit is meter.
Displacement:
The shortest distance between the initial and final positions of any object during motion is called displacement. The displacement of an object in a given time can be positive, zero or negative.
It is a vector quantity. Its SI unit is meter
Displacement = (Final position -Initial position)
$\Delta X = (X2-X1)$
Speed:
The time rate of change of position of the object in any direction is called speed of the object.

Speed (v) = Distance travelled (s) / Time taken (t), v =s/t
Its unit is m/s. It is a scalar quantity. Its dimensional formula is $[M^0 L^1 T^{-1}]$.

Average Speed

The ratio of the total distance travelled by the object to the total time taken is called average speed of the object.
Average speed = Total distanced travelled / Total time taken
If a particle travels distances s1, s2, s3 , … with speeds v1, v2, v3, …, then
Average speed = s1 + s2 + s3 + ….. / (s1 / v1 + s2 / v2 + s3 / v3 + …..)

If particle travels equal distances (s1 = s2 = s) with velocities v1 and v2, then
Average speed = 2 v1 v2 / (v1 + v2)

If a particle travels with speeds v1, v2, v3, …, during time intervals t1, t2, t3,…, then
Average speed = v1t1 + v2t2 + v3t3 +… / t1 + t2 + t3 +….

If particle travels with speeds v1, and v2 for equal time intervals, i.e., t1 = t2 = t3, then
Average speed = (v1 + v2) / 2

When a body travels equal distance with speeds v1 and v2, the average speed (v) is the harmonic mean of two speeds.

2 / v = 1 / v1 + 1 / v2

Instantaneous Speed

When an object is travelling with variable speed, then its speed at a given instant of time is called its instantaneous speed.
Instantaneous speed $V_i = \lim_{\Delta t \to 0} \frac{ds}{dt}$

Velocity:

The rate of change of displacement of an object in a particular direction is called its velocity.
Velocity =Displacement / Time taken
Its unit is m/s.
Its dimensional formula is $[M^o LT^{-1}]$.
It is a vector quantity, as it has both, the magnitude and direction.
The velocity of an object can be positive, zero and negative.
Types of velocity:
(a) Uniform Velocity
If an object undergoes equal displacements in equal intervals of time, then it is said to be moving with a uniform velocity.
(b) Non-uniform or Variable Velocity
If an object undergoes unequal displacements in equal intervals of time, then it is said to be moving with a non-uniform or variable velocity.
(c) Relative Velocity
Relative velocity of one object with respect to another object is the time rate of change of relative position of one object with respect to another object.
Relative velocity of object A with respect to object B
$V_{AB} = V_A – V_B$
(i)When two objects are moving in the same direction,

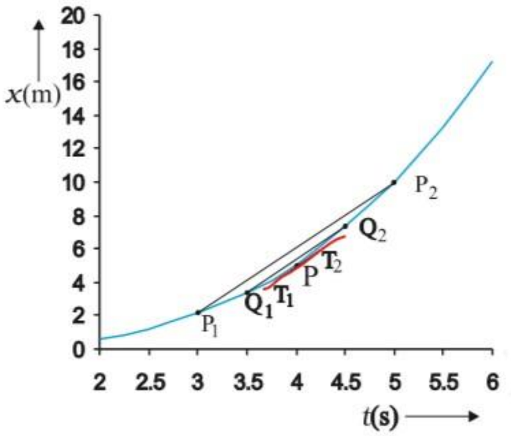
INSTANTANEOUS VELOCITY

The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

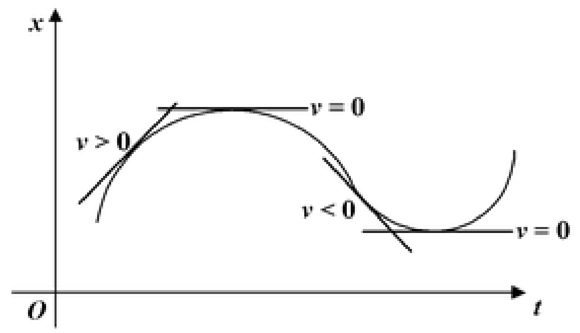
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$= \frac{dx}{dt}$$

Graphically,



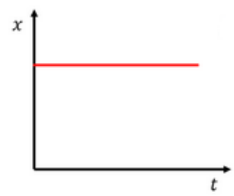
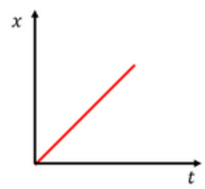
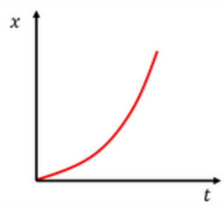
Determining velocity from position-time graph. Velocity at t = 4 s is the slope of the tangent to the graph at that instant.

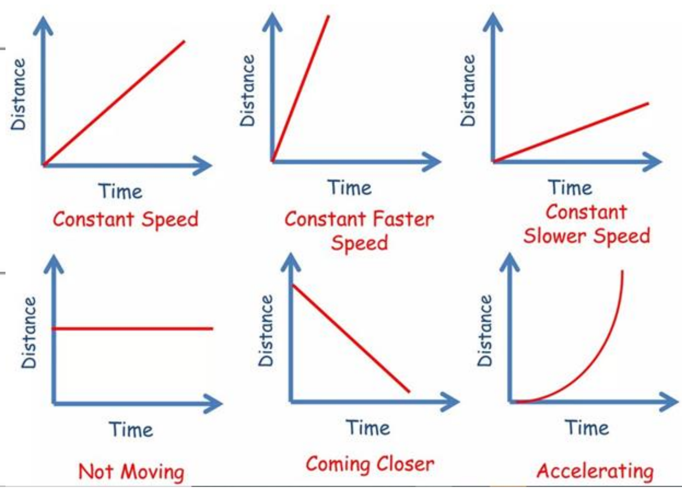


The sign of the instantaneous velocity depends on the slope of the tangent.

It should be noted that though average speed over a finite interval of time is greater or equal to the magnitude of the average velocity, instantaneous speed at an instant is equal to the magnitude of the instantaneous velocity at that instant.

Distance- Time Graphs:-

1) At rest: i.e. the slope of $x - t$ (velocity) $= \tan 0° = 0$ $\Rightarrow v = 0$	
2) For uniform motion: (velocity v = constant) $x = kt$, k is a positive constant Slope $= \frac{dx}{dt} = \frac{d}{dt}(kt) = k$ i.e. velocity is constant.	
3) Uniformly accelerated motion (Acceleration a is constant) $x = \frac{1}{2}at^2$ Acceleration $= \frac{d^2x}{dt^2} = a$ (Constant)	



ACCELERATION

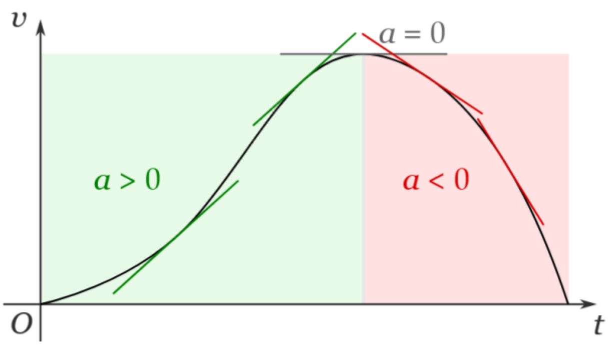
The average acceleration **a** over a time interval is defined as the change of velocity divided by the time interval.

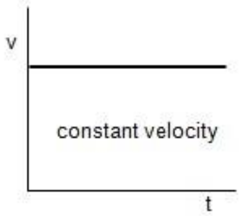
$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

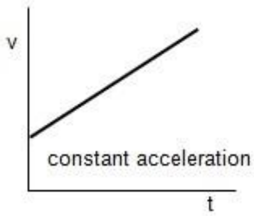
Instantaneous acceleration is defined in the same way as the instantaneous velocity:

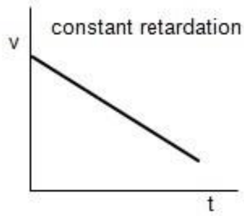
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

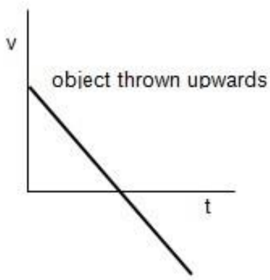
- The acceleration at an instant is the slope of the tangent to the v–t curve at that instant.
- So, we can easily determine when the acceleration is positive, negative, and zero, just by looking at the angle θ at different points with x-axis on a velocity vs time graph:

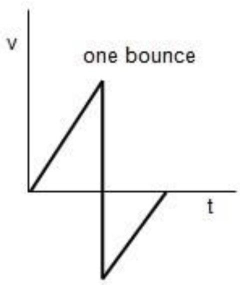


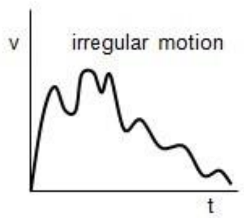










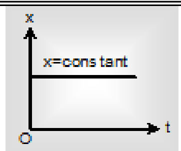
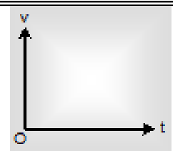
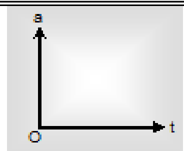
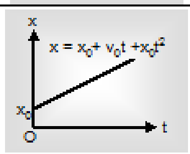
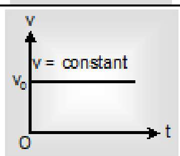
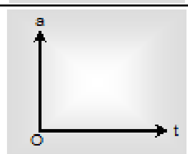
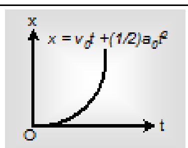
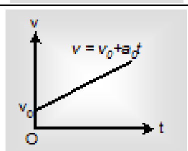
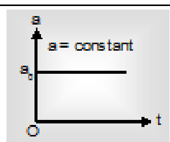
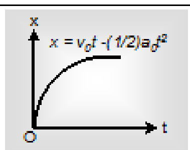
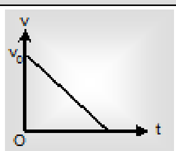
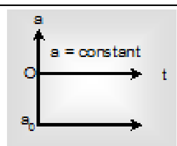


- Its unit is m/s^2
- Its dimensional formula is $[M^oLT^{-2}]$.
- It is a vector quantity.
- Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform while negative acceleration (retardation) means velocity is decreasing with time.
- If a particle is accelerated for a time t_1 with acceleration a_1 and for a time t_2 with acceleration

a_2, then average acceleration
$a_{av} = (a_1t_1 + a_2t_2)/(t_1 + t_2)$

Note: that the x-t, v-t, and a-t graphs shown in several figures in this chapter have sharp kinks at some points implying that the functions are not differentiable at these points. In any realistic situation, the functions will be differentiable at all points and the graphs will be smooth.

What this means physically is that acceleration and velocity cannot change values abruptly at an instant. Changes are always continuous.

	Displacement(x)	Velocity(v)	Acceleration (a)
a. At v=0;			
b. Motion with constant velocity			
c. Motion with constant acceleration			
d. Motion with constant deceleration			

An interesting feature of a velocity-time graph for any moving object is that **the area under the curve represents the displacement over a given time interval.**

KINEMATIC EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

Algebraic Method:

For uniformly accelerated motion, we can derive some simple equations that relate displacement (s), time taken (t), initial velocity (u), final velocity (v) and acceleration (a).

Equation- 1,

Since, uniform acceleration a : $a = v - u / t$

So, ***v = u + at*** -------------- ***(i)***

Equation- 2

Since $v_{av} = ½ (v_0 + v)$ { for constant acceleration) and distance $s = v_{av} \times t$

So, $s = \frac{u+v}{2} \times t$ => $s = \frac{(u+(u+at))}{2} \times t$ by substituting the value of v

Therefore $s = (u + \frac{1}{2}at) \times t$ => $\boldsymbol{s = ut + \frac{1}{2}at^2}$ ---------------- ***(ii)***

Equation- 3

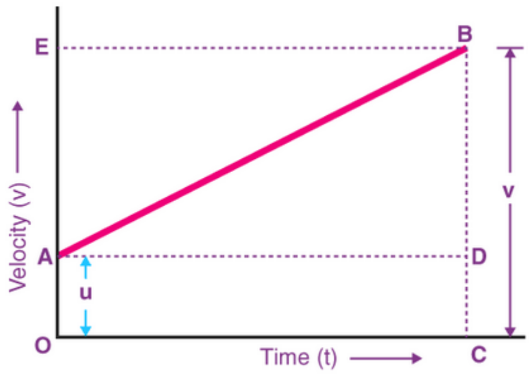
As, $v_{av} = ½ (v_0 + v)$ { for constant acceleration) and distance $s = v_{av} \times t$

So, s = ½ (v+ u) { (v - u) / a } by substituting t= {(v – u) / a } from equation (i)

Therefore, $s = v^2 – u^2 / 2a$ => $\boldsymbol{v^2 – u^2 = 2as}$ ----------------- ***(iii)***

Graphical Method

- Slope of the v-t graph represents acceleration, and
- the area under the curve in the fig.is equivalent to the displacement covered



1. From the graph, we know that
v-u = BD (*Equation 1*)
Now, since the slope of a velocity-time graph is equal to acceleration ***a***.
So,
a = slope of line AB
a = BD/AD
Since ***AD = AC = t***, the above equation becomes:
BD = at (*Equation 2*)
Now, combining *Equation 1 & 2*, the following is obtained:

v-u = at -------------------- ***(i)***

2. From the graph above, we can say that
Distance travelled (s) = Area of figure OABC = Area of rectangle OADC + Area of triangle ABD

$$s = (\frac{1}{2} \times AD \times BD) + (OA \times OC)$$

As OA=u and OC=t, the above equation becomes,

$$s = (\frac{1}{2} \times AD \times BD) + (u \times t)$$

As BD =at (from the graphical derivation of 1st equation of motion), the equation becomes,

$$s = \frac{1}{2} \times t \times at + ut$$

On further simplification, the equation becomes

$\boldsymbol{s = ut + \frac{1}{2}at^2}$ ---------------------- ***(ii)***

3. From the graph, we can say that
The total distance travelled, ***s*** is given by the **Area of trapezium OABC**.
Hence,
s = ½ × (Sum of Parallel Sides) × Height
s = 1/2 x (OA + CB) x OC
Since, **OA = u**, **CB = v**, and **OC = t**
The above equation becomes
s = 1/2 x (u+v) x t
Now, since **t = (v – u)/ a**
The above equation can be written as:
s = ½ x ((u+v) × (v-u))/a
Rearranging the equation, we get
s = ½ x (v+u) × (v-u)/a
s = $(v^2-u^2)/2a$
Third equation of motion is obtained by solving the above equation:

$$v^2 = u^2+2as \quad \text{-----------------------}(iii)$$

Calculus Method:

1. Since acceleration is the rate of change of velocity, it can be mathematically written as: $a = \frac{dv}{dt}$

   Rearranging the above equation, we get : $adt = dv$

   Integrating both the sides, we get : $\int_0^t adt = \int_u^v dv$ => $at = v - u$ => $\boldsymbol{v = u + at}$ ---------- ***(i)***

2. Velocity is the rate of change of displacement.

   Mathematically, this is expressed as $v = \frac{ds}{dt}$
   Rearranging the equation, we get, $ds = vdt$
   Substituting the first equation of motion in the above equation, we get

$$ds = (u + at)dt$$
$$ds = (u + at)dt = (udt + atdt)$$

   Integrating both sides, we get

$$\int_0^s ds = \int_0^t udt + \int_0^t atdt$$

   On further simplification, the equation becomes:

$$\boldsymbol{s = ut + \frac{1}{2}at^2} \text{ ------------------- } \textit{(ii)}$$

3. We know that acceleration is the rate of change of velocity and can be represented as:

$$a = \frac{dv}{dt} \ldots.(1)$$

   We also know that velocity is the rate of change of displacement and can be represented as:

$$v = \frac{ds}{dt} \ldots.(2)$$

   Cross multiplying (1) and (2), we get

$$a\frac{ds}{dt} = v\frac{dv}{dt}$$

$$\int_0^s ads = \int_u^v vdv$$

$$as = \frac{v^2-u^2}{2} \qquad \Rightarrow \quad \mathbf{v^2 = u^2 + 2as} \text{ --------------} \textbf{\textit{(iii)}}$$