Physics

# Chapter 1- UNITS AND MEASUREMENT

Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called unit.

Although the number of physical quantities appears to be very large, we need only a limited number of units (Fundamental units) for expressing all the physical quantities, since they are interrelated with one another.

**THE INTERNATIONAL SYSTEM OF UNITS**

In earlier time scientists of different countries were using different systems of units for measurement. Three such systems, the CGS, the FPS (or British) system and the MKS system were in use extensively till recently.

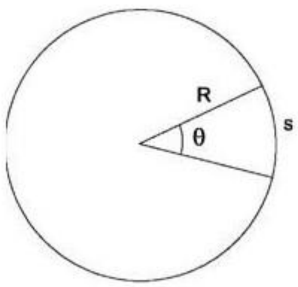
The system of units which is at present internationally accepted for measurement is the Système Internationale d' Unites (French for International System of Units), abbreviated as SI.

**Table 1. Present SI base quantities, base units, and definitions**

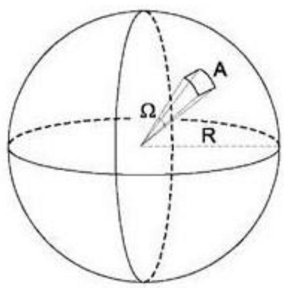
| Base quantity | Base unit | Definition |
|---|---|---|
| Time | second | The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. |
| Length | meter | The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second. |
| Mass | kilogram | The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. |
| Electric current | ampere | The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newton per meter of length. |
| Thermodynamic temperature | kelvin | The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. |
| Amount of substance | mole | The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12; the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles. |
| Luminous intensity | candela | The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \times 10^{12}$ hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. |

Besides the seven base units, there are **two more units** that are defined for
(a) plane angle dθ, and (b) solid angle dΩ.

The unit for plane angle is radian with the symbol rad and the unit for the solid angle is steradian with the symbol sr. Both these are dimensionless quantities.




$\theta = \frac{s}{R}$ radians




$\Omega = \frac{A}{R^2}$ steradians (sr)

**Some units retained for general use (Though outside SI)**

| Name | Symbol | Value in SI Unit |
|---|---|---|
| minute | min | 60 s |
| hour | h | 60 min = 3600 s |
| day | d | 24 h = 86400 s |
| year | y | 365.25 d = 3.156 × $10^7$ s |
| degree | ° | 1° = (π/ 180) rad |
| litre | L | I $dm^3 = 10^{-3} m^3$ |
| tonne | t | $10^3$ kg |
| carat | c | 200 mg |
| bar | bar | 0.1 MPa = $10^5$ Pa |
| curie | Ci | 3.7 × $10^{10} s^{-1}$ |
| roentgen | R | 2.58 × $10^{-4}$ C/kg |
| quintal | q | 100 kg |
| barn | b | 100 $fm^2 = 10^{-28} m^2$ |
| are | a | 1 $dam^2 = 10^2 m^2$ |
| hectare | ha | 1 $hm^2 = 10^4 m^2$ |
| standard atmospheric pressure | atm | 101325 Pa = 1.013 × $10^5$ Pa |

**SIGNIFICANT FIGURES:**

Every measurement involves errors. Thus, the result of measurement should be reported in a way that indicates the precision of measurement. The reliable digits plus the first uncertain digit are known as significant digits or significant figures.

Example: - The length of an object reported after measurement to be 287.5 cm has four significant figures, the digits 2, 8, 7 are certain while the digit 5 is uncertain.

The rules for determining the number of significant figures: -

- The precision of measurement which depends on the least count of the measuring instrument.
- A choice of change of different units does not change the number of significant digits or figures in a measurement.
  For example, the length 2.308 cm has four significant figures. But in different units, the same value can be written as 0.02308 m or 23.08 mm or 23080 µm. All these numbers have the same number of significant figures (digits 2, 3, 0, 8), namely four.
  This shows that the location of decimal point is of no consequence in determining the number of significant figures.

• All the non-zero digits are significant.

• All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.

• If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant. [In 0.00 2308, the underlined zeroes are not significant].

• The terminal or trailing zero(s) in a number without a decimal point are not significant.

[Thus 123 m = 12300 cm = 123000 mm has three significant figures, the trailing zero(s) being not significant.] However, you can also see the next observation.

• The trailing zero(s) in a number with a decimal point are significant. [The numbers 3.500 or 0.06900 have four significant figures each.]

- Every number is expressed as a × $10^b$, where a is a number between 1 and 10, and b is any positive or negative exponent (or power) of 10.
- For a number greater than 1, without any decimal, the trailing zero(s) are not significant.
- For a number with a decimal, the trailing zero(s) are significant.
- The digit 0 conventionally put on the left of a decimal for a number less than 1 (like 0.1250) is never significant. However, the zeroes at the end of such number are significant in a measurement.
- The multiplying or dividing factors which are neither rounded numbers nor numbers representing measured values are exact and have infinite number of significant digits. For example in r = d/2 or s = 2πr, the factor 2 is an exact number and it can be written as 2.0, 2.00, or 2.0000 as required.

**Rules for Arithmetic Operations with Significant Figures**

1. In multiplication or division, the final result should retain as many **significant figures** as are there in the original number with the least significant figures. Example: density = 4.237 g/ **2.52** $cm^3$ = **1.69** g/$cm^3$ (3 significant figures)
2. In addition, or subtraction, the final result should retain as many **decimal places** as are there in the number with the least decimal places. Example: 436.32 g +**227.2** g +0.301 g = 663.821 g. But the final result should, therefore, be rounded off to **663.8** g. Similarly 0.307 m – 0.304 m = 0.003 m = 3 ×$10^{-3}$ m.

Note that we should not use the rule (1) applicable for multiplication and division and write 664 g as the result in the example of addition and 3.00 × 10–3 m in the example of subtraction.

## Rounding off the Uncertain Digits

The rule by convention is that

the preceding digit is raised by 1 if the insignificant digit to be dropped (the underlined digit in this case) is more than 5, and is left unchanged if the latter is less than 5.

But what if the number is 2.745 in which the insignificant digit is 5. Here, the convention is that

if the preceding digit is even, the insignificant digit is simply dropped and, if it is odd, the preceding digit is raised by 1. Then, the number 2.745 rounded off to three significant figures becomes 1.74. On the other hand, the number 2.735 rounded off to three significant figures becomes 1.74 since the preceding digit is odd.

## Rules for Determining the Uncertainty in the Results of Arithmetic Calculations

1. If the length and breadth of a thin rectangular sheet are measured, using a metre scale as 16.2 cm and, 10.1 cm respectively, there are three significant figures in each measurement. It means that the length l may be written as l = 16.2 ± 0.1 cm
   = 16.2 cm ± 0.6 %. Similarly, the breadth b may be written as
   b = 10.1 ± 0.1 cm = 10.1 cm ± 1 %
   Then, the error of the product of two (or more) experimental values, using the combination of errors rule, will be
   l b = 163.62 $cm^2$ + 1.6%
   = 163.62 + 2.6 $cm^2$
   This leads us to quote the final result as
   l b = 164 + 3 $cm^2$ , Here 3 $cm^2$ is the uncertainty or error in the estimation of area of rectangular sheet.

2. If a set of experimental data is specified to n significant figures, a result obtained by combining the data will also be valid to n significant figures.
   Exception: Uncertainties in subtraction or addition combine in a different fashion (smallest number of decimal places rather than the number of significant figures in any of the number added or subtracted).

3. The relative error of a value of number specified to significant figures depends not only on n but also on the number itself.
   For example, the accuracy in measurement of mass 1.02 g is ± 0.01 g whereas another measurement 9.89 g is also accurate to ± 0.01 g. The relative error in 1.02 g is = (± 0.01/1.02) × 100 % = ± 1% Similarly, the relative error in 9.89 g is = (± 0.01/9.89) × 100 % = ± 0.1 %
4. Intermediate results in a multi-step computation should be calculated to one more significant figure in every measurement than the number of digits in the least precise measurement.
   For example, the reciprocal of 9.58, calculated (after rounding off) to the same number of significant figures (three) is 0.104, but the reciprocal of 0.104 calculated to three significant figures is 9.62. However, if we had written 1/9.58 = 0.1044 and then taken the reciprocal to three significant figures, we would have retrieved the original value of 9.58.

## DIMENSIONS OF PHYSICAL QUANTITIES

The nature of a physical quantity is described by its dimensions.

All the physical quantities represented by derived units can be expressed in terms of some combination of seven fundamental or base quantities.

**Length** has the dimension [L], **mass** [M], **time** [T], **electric current** [A], **thermodynamic temperature** [K], **luminous intensity** [cd], and **amount of substance** [mol].

Note that using the square brackets [ ] round a quantity means that we are dealing with 'the dimensions of' the quantity.

In **mechanics,** all the physical quantities can be written in terms of the dimensions **[L], [M] and [T]**

### DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the ***dimensional formula*** of the given physical quantity.

For example, the dimensional formula of the volume is [M° L3 T°], and that of speed or velocity is [M° L T-1].

An equation obtained by equating a physical quantity with its dimensional formula is called the ***dimensional equation*** of the physical quantity.

For example, Volume [V] = $[M^0 L^3 T^0]$ and Speed [v] = $[M^0 L T^{-1}]$

### DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

1. **Checking the Dimensional Consistency of Equations**

**The principle of homogeneity of dimensions: -** The magnitudes of physical quantities may be added together or subtracted from one another only if they have the same dimensions.

Thus, velocity cannot be added to force, or an electric current cannot be subtracted from the thermodynamic temperature.

- U*seful in checking the correctness of an equation;* If the dimensions of all the terms are not same, the equation is **wrong.**
- However, the dimensional consistency does not guarantee **correctness** of equations. It is uncertain to the extent of dimensionless quantities or functions. Trigonometric, logarithmic and exponential functions must be dimensionless. A pure number, ratio of similar physical quantities, such as angle as the ratio (length/length), refractive index etc., has no dimensions.

2. **Deducing Relation among the Physical Quantities**

For this we should know the dependence of the physical quantity on other quantities (upto three physical quantities or linearly independent variables) and consider it as a product type of the dependence. Let us take an example.

Suppose that the period of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g).

$T = k\, l^x g^y m^z$

where k is dimensionless constant and x, y and z are the exponents.

By considering dimensions on both sides, we have

$[L^0 M^0 T] = [L^1]^x [L T^{-2}]^y [M]^z$

$= L^{x+y} T^{-2y} M^z$

On equating the dimensions on both sides, we have $x + y = 0$; $-2y = 1$; and $z = 0$

So that $x=1/2$, $y= -1/2$ and $z=0$

Then, $T = k\, l^{1/2} g^{-1/2} = k \sqrt{l/g}$

Note that value of constant k cannot be obtained by the method of dimensions.

Drawbacks:

- Dimensionless constants cannot be obtained by this method
- The method of dimensions can only test the dimensional validity, but not the exact relationship between physical quantities in any equation.
- It does not distinguish between the physical quantities having same dimensions.

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